E. Morera Theorem

Ref: Complex Variables by James Ward Brown and Ruel V. Churchil

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Theorem : 2 (E.morera 1856-1909)

Let **f** be continuous on a domain D. If $\int_C f(z)dz = 0$ …… **(6)** for every closed contour C lying in D, then f is analytic throughout D.

Proof:

Given: $\int_C f(z)dz = 0$ for every closed contour C in D .

$\Rightarrow f$ has an anti derivative in D (by a theorem in section **42**)

$\Rightarrow$there exists an analytic function F such that $F'(z)=f(z)$ at each point in D.

We know that F is analytic $\Rightarrow F'$ is analytic in D. $\Rightarrow f$ is analytic is D. ■

Remark:

1. If the point lies outside of the given region then $\frac{1}{2\pi i}\int_C \frac{f(z)dz}{z-z_0} = 0$ (by Cauchy Theorem).

2. π radians = 180 degrees. i.e.) $\frac{22}{7}$ radians = 180 degrees.

   In working out problems, put $\pi = \frac{22}{7} = 3.14$. ■

Problem :

Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2 \text{ and } y = \pm 2$. Find $\int_C \frac{(\tan(z/2))dz}{(z-x_0)^2}$ $(-2 < x_0 < 2)$.

Solution :

Since $-2 < x_0 < 2$, x_0 lies inside the given square. So

$$\int_C \frac{\left(\tan\left(z/2\right)\right)}{\left(z-x\right)^2}\,dz = 2\pi i\left[f'\left(z\right)\right] \text{ at } z = x_0$$

$$= 2\pi i\left[\frac{d}{dz}\left(\tan\left(\frac{z}{2}\right)\right)\right] \text{ at } z = x_0$$

$$= 2\pi i\left[\frac{1}{2}\sec^2\left(\frac{z}{2}\right)\right] \text{ at } z = x_0$$

$$= \pi i \sec^2\left(\frac{x_0}{2}\right).$$ ■

Problem:

Find $\int_C \frac{z\,dz}{z-2}$ where C is $|z|=1$

Solution : 2 lies outside $|z|=1$ and so $\frac{z}{z-2}$ is analytic inside C. So $\int_C \frac{z\,dz}{z-2} = 0$. ∎